MATH PROBLEM SOLVING
FOR PRIMARY ELEMENTARY STUDENTS
WITH DISABILITIES

ABOUT THE AUTHOR

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MATH PROBLEM SOLVING FOR PRIMARY SCHOOL STUDENTS WITH DISABILITIES

This brief is part of a series on teaching primary, upper elementary, and middle school students with disabilities how to solve mathematical word problems. The skills and strategies needed for successful mathematical problem solving develop from the preschool years, when children acquire a basic conceptual understanding of the base 10 numerical system. During these early years, they typically develop the "number sense" needed to process and manipulate numerical information. In primary school, children continue to acquire mathematical knowledge and skills and are exposed to a variety of math problem types requiring addition and subtraction operations. Students in upper elementary school continue to apply and refine the skills and strategies necessary to solve real life mathematics problems. By middle school, students should be able to apply mathematical problem-solving skills and strategies effectively and efficiently in school, at home, and in the community.

Most children enter kindergarten with a fairly developed sense of numbers (Case, 1998, as cited in Gersten & Chard, 1999). Children with number sense can invent their own procedures for doing math, they can represent numbers in many different ways, and they can recognize benchmark numbers and number patterns. They also seem to understand the magnitude of numbers and recognize when a number's order of magnitude does not make sense. They are able to talk about quantities without actually performing operations. Children with well-developed number sense are comfortable with mathematics and enjoy solving math problems.

This brief deals specifically with math problem solving for primary school students. Many students in kindergarten through grade 3, especially students with learning disabilities (LD), have difficulty learning how to solve math word problems because they often do not have the necessary conceptual bases. From kindergarten through third grade, both children who have reading skills and those who must be read to are expected to be able to solve problems. The following are examples of second and third grade problems:

- Charlie has 15 stickers. His friend Tony gave him 6 more stickers. How many stickers does Charlie have altogether?
- Sara has 14 balls. Amy has 5 balls. How many more balls does Sara have?

For students who can read, most textbooks are not very helpful when it comes to teaching students how to solve math problems. They typically provide a four-step formula: (a) read the problem, (b) decide what to do, (3) compute, and (4) check your answer. "Read" the problem for understanding is the first step. Understanding involves a representation of the relationship between numbers, words, and symbols in the problem. This representation provides the basis for deciding what to do to solve the problem. From early on, most students acquire the skills and strategies needed to "read the problem" and "decide what to do" to solve it. Many students with LD or other cognitive impairments, however, do not easily acquire these skills and strategies. Therefore, they need explicit instruction in mathematical problem-solving skills and strategies to solve problems in their math textbooks and in their daily lives.

The following frequently asked questions provide the framework for this brief and the other briefs in this series brief.

- What is mathematical problem solving?
- How do good problem solvers solve math problems?
- Why is it so difficult to teach students to be good math problem solvers?
- What is the content of math problem-solving instruction?
- What are effective instructional procedures for teaching math problem solving?

Several validated practices for teaching young children math problem solving are described, and a sample lesson is provided. Additionally, specific adaptations and accommodations are provided for students with other types of cognitive disabilities such as traumatic brain injury (TBI) and attention deficit hyperactivity disorder (ADHD).

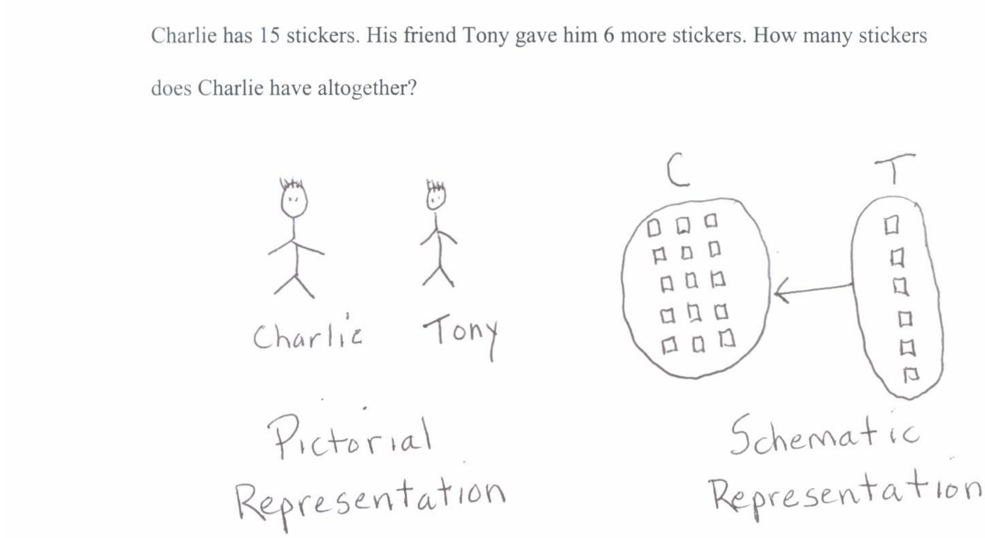
What is mathematical problem solving?

Mathematical problem solving is a complex cognitive activity involving a number of processes and strategies. Problem solving has two stages: problem representation and problem execution. Successful problem solving is not possible without first representing the problem. Appropriate problem representation indicates the problem solver has understood the problem, and it guides the student toward the solution plan. Students who have difficulty representing math problems will have difficulty solving them.

Students in primary school, kindergarten through grade 3, are acquiring declarative and procedural knowledge (i.e., math facts and math operations) in addition and subtraction. They are expected to be able to apply this knowledge in solving "simple" addition and subtraction word problems, but these "simple" problems are more complicated than at first glance. There are actually four addition and subtraction problem types: change problems, compare problems, equalize problems, and combine problems. Within the change, compare, and equalize problem types, there are six variations of the position of the unknown quantity. Within the combine problem type, there are two variations. Therefore, students with mathematical difficulties need explicit instruction in 20 different variations of "simple" primary-level word problems. The position of the unknown quantity has a major influence on the difficulty level of the problem. So, in line with good principles of instruction for students with disabilities, problem types should be introduced beginning with the easiest problem type, and, after mastery, move to the next level of difficulty (see García, Jiménez, & Hess, in press, for a taxonomy of the difficulty levels of the four types of addition and subtraction problems for students with mathematical difficulties).

Teaching mathematical problem solving is a challenge for many teachers, many of whom rely almost exclusively on mathematics textbooks to guide instruction. Unfortunately, most basal series do not provide the teacher with the tools to teach problem solving. Instruction for primary children with math learning disabilities must progress gradually from the concrete to the abstract. That is, symbolic representation may not be possible for these children without explicit instruction that begins with manipulatives and other tactile materials that are concrete in nature (e.g., abacus, base ten blocks, Cuisenaire rods) that will help students move from a concrete to a more symbolic, schematic level. In other words, teachers must provide systematic, progressive, and scaffolded instruction that considers the students' prior knowledge and cognitive strengths and weaknesses.

Students who have difficulty solving math word problems usually do not construct a representation of the problem that considers the relationships among the problem components and, as a result, they do not understand the problem and have no clue about a plan to solve it. So, it is not simply a matter of "drawing a picture or making a diagram;" rather, it is the type of picture or diagram that is important. Effective visual representations, whether with manipulatives, with paper and pencil, or in one's imagination, show the relationships among the problem parts. These are called schematic representations (van Garderen & Montague, 2003). Poor problem solvers tend to make immature representations that are more pictorial than schematic in nature. The illustration below shows the difference between a pictorial and a schematic representation of the mathematical problem presented earlier in the brief.



Other cognitive processes and strategies needed for successful mathematical problem solving include paraphrasing the problem, which is a comprehension strategy, hypothesizing or setting a goal and making a plan to solve the problem, estimating or predicting the outcome, computing or doing the arithmetic, and checking to make sure the plan was appropriate and the answer is correct (Montague, 2003; Montague, Warger, & Morgan, 2000). Mathematical problem solving also requires metacognitive or self-regulation strategies. Students with LD are notoriously poor self-regulators. In this developmental period in particular, it is imperative that they be explicitly taught how to self-instruct (tell themselves what to do), self-question (ask themselves questions), and self-monitor (check themselves as they solve the problem).

What do good problem solvers do?

Good problem solvers use a variety of processes and strategies as they read and represent the problem before they make a plan to solve it (Montague, 2003). First, they **READ the problem for understanding.** As they read, they use comprehension strategies to translate the linguistic and numerical information in the problem into mathematical notations. For example, good problem solvers may read the problem more than once and may reread parts of the problem as they progress and think through the problem. They use self-regulation strategies by asking themselves if they understood the problem. They **PARAPHRASE the problem by putting it into their own words.** They identify the important information and may even underline parts of the problem. Good problem solvers ask themselves what the question is and what they are looking for. **VISUALIZING or drawing a picture or diagram** means developing a schematic representation of the problem so that the picture or image reflects the relationships among all the important problem parts. Using both verbal translation and visual representation, good problem solvers not only are guided toward understanding the problem, but they are also guided toward developing a plan to solve the problem. This is the point at which students decide what to do to solve the problem. They have represented the problem and they are now ready to develop a solution path. They **HYPOTHESIZE** by thinking about logical solutions and the types of operations and number of steps needed to solve the problem. They may write the operation symbols as they decide on the most appropriate solution path and the algorithms they need to carry out the plan. They ask themselves if the plan makes sense given the information they have. Good problems solvers usually **ESTIMATE or predict the answer** using mental calculations, or may even quickly use paper and pencil as they round the numbers up and down to get a "ballpark" idea. They are now ready to **COMPUTE.** So they tell themselves to do the arithmetic and then compare their answer with their estimate. They also ask themselves if the answer makes sense and if they have used all the necessary symbols and labels such as dollar signs and decimals. Finally, they **CHECK** to make sure they used the correct procedures and that their answer is correct.

These processes and strategies are developmental in nature and reach maturation at different stages. For example, visualization matures in most learners sometime between the ages of 8 and 11. Primary school students with math disabilities will need cues and prompts and other ongoing supports as they learn how to solve problems and also during practice sessions. Typically, these children have memory and self-regulation problems, so instruction must provide compensatory supports to mediate the cognitive problems that interfere with problem solving. Many children will require cues and prompts not only during instruction but also following instruction to maintain the problem-solving skills and strategies they have learned.

Why is it so difficult to teach students to solve math problems?

Students who are poor mathematical problem solvers, as most students with LD are, do not process problem information effectively or efficiently. They lack or do not apply the resources needed to complete this complex cognitive activity. Generally, these students also lack metacognitive or self-regulation strategies that help successful students understand, analyze, solve, and evaluate problems. To help these students become good problem solvers, teachers must understand and teach the cognitive processes and metacognitive strategies that good problem solvers use. This is the **CONTENT** of math problem-solving instruction. Teachers must also use instructional **PROCEDURES** that are research-based and have proven effective. These procedures are the basis of **COGNITIVE STRATEGY INSTRUCTION**, which has been demonstrated to be one of the most powerful interventions for students with LD (Swanson, 1999).

What is the content of math problem-solving instruction?

The previous sections described the content of math problem-solving instruction as the cognitive processes and metacognitive strategies that good problem solvers use to solve mathematical problems. Students learn how to use these processes and strategies not only effectively, but efficiently as well. For primary school students, teaching needs to be systematic. The focus should be on reading, paraphrasing, and visualizing. Manipulatives are essential. A representation using manipulatives is developed with the support of the teacher. Based on the three-dimensional representation, a schematic representation using paper and pencil is formed. Eventually the representation is transformed into a symbolic mathematical representation using mathematical notation.

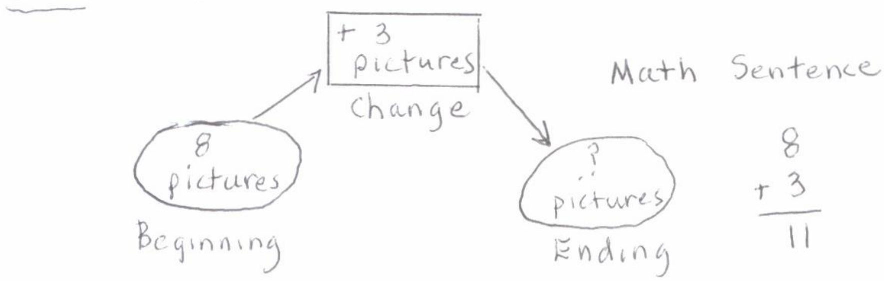
Jitendra, Griffin, Haria, Leh, Adams, and Kaduvetoor (2005) have developed a procedure called schema-based strategy instruction for teaching primary school students how to solve change, group, and compare problems. They use a four-step process.

- *Find* the problem type.
- *Organize* the information in the problem using the diagram.
- *Plan* to solve the problem.
- *Solve* the problem.

For each problem, a diagram and self-regulation checklist was provided. See the below figure for an example of the diagram and checklist for a change problem.

Change Problem

Tammy likes to paint pictures. She has painted 8 pictures so far. If she paints 3 more pictures, how many will she have?



Change Problem Checklist

Step 1. Find the problem type.
- Did I read and retell the problem?
- Did I ask if it is a change problem? (Did I look for the beginning, change, and ending? Do they all describe the same thing?)

Step 2. Organize the information using the change diagram.
- Did I underline the label that describes the beginning, change, and ending and write in the label in the diagram?
- Did I underline the important information, circle the numbers, and write the numbers in the diagram?
- Did I write a "?" for what must be solved? (Did I find my question sentence?)

Step 3. Plan to solve the problem.
- Do I add or subtract?
- Did I write the math sentence?

Step 4. Solve the problem.
- Did I solve the math sentence?
- Did I write the complete answer?
- Did I check if the answer makes sense?

What are effective instructional procedures for teaching math problem solving?

Explicit Instruction

Explicit Instruction, the basis of cognitive strategy instruction, incorporates research-based practices and instructional procedures such as cueing, modeling, verbal rehearsal, and feedback. The lessons are highly organized and structured. Appropriate cues and prompts are built in as students learn and practice the cognitive and metacognitive processes and strategies. Each student is provided with immediate, corrective, and positive feedback on performance. Overlearning, mastery, and automaticity are the goals of instruction. Explicit instruction allows students to be active participants as they learn and practice math problem-solving processes and strategies. This approach emphasizes interaction among students and teachers.

Through an extensive and statistical review of the intervention studies conducted over 20 years with students with LD, Swanson (1999) identified the following eight components of effective strategy instruction. They are described as they would be used in teaching mathematical problem solving.

Sequencing and Segmenting

Sequencing and segmenting means breaking the task into component subparts, providing short activities, and synthesizing the parts into a whole. For example, each cognitive process/self-regulation strategy routine is taught consecutively, beginning with reading the problem as a necessary first step for solving the problem. Students are taught to read the problem and then to ask themselves if they understood it. They are then taught to go back and reread it or read parts until they decide they understand it. At the beginning of instruction, the teacher models the process and provides plenty of step-by-step cues and prompts as students practice. Eventually these cues and prompts are phased out. After students know what to do when they read math problems, they learn how to paraphrase or retell problems. Students learn the paraphrasing routine, which is then added to the reading routine. Subsequently, students have mastered a sequence of two important processes for solving mathematical problems.

Drill-repetition and Practice-review

This component includes progress checks to measure skill mastery, sequenced review, repeated practice, distributed review and practice using the same or similar practice problems, and ongoing and positive feedback. For example, the paraphrasing routine is taught and then students practice on their own or with peers.

PARAPHRASE (your own words)	
Say:	Underline the important information. Put the problem in my own words.
Ask:	Have I underlined the important information? What is the question? What am I looking for?
Check:	That the information goes with the question.

After the teacher models the routine and guides the students as they go through the routine, they are provided with practice until the routine becomes automatic. As they learn how to paraphrase math word problems, they can evaluate themselves using a checklist, and plot their improvement on a graph.

Directed Questioning and Responses

Cognitive strategy instruction uses a guided discussion technique to promote active teaching and learning. Students are engaged from the very beginning through an initial discussion of the importance of mathematical problem solving. With the teacher, they set individual performance goals and make a commitment to becoming a better problem solver. Teachers ask both “process-related” and “content-related” questions. Students are directed by the teacher to ask questions. Students are also taught when and how to ask for help.

Control Difficulty or Processing Demands of the Task

Arrange tasks from easy to difficult. The teacher provides simplified demonstrations, necessary assistance, appropriate cues and prompts, and guided discussion. For primary school students, math problem-solving instruction should start with one-step change addition problems in which the “ending” is unknown (see Figure 2). When students have mastered the problem solving routine with problems of this type, they can progress to one-step change problems in which the change is unknown. They can then progress to change problems in which the beginning is unknown, and so forth.

Technology

Technology extends beyond calculators and computers to include structured text, diagrams, flow charts, structured curricula, scripted lessons, and video demonstrations. Students who are learning to be better math problem solvers should be taught how to use calculators to facilitate computation after their understanding of math facts for addition and subtraction have been mastered.

Group Instruction

Students with LD who have math problem solving difficulties should be taught in small groups (5-8 students) to maximize teacher and student interaction. Interaction between teachers and students and among peers is the cornerstone of cognitive strategy instruction. Cognitive strategy instruction is intensive and time-limited.

Supplements to Teacher and Peer Involvement

Children with second and third grade reading skills can be given cue cards to study for homework as they memorize and learn the various problem-solving processes and self-regulation strategies. For example, using the Jitendra, et al., strategy for change problems, students first learn to “find the problem” type by reading the problem, retelling the problem, and asking themselves if it is a change problem. They check the box on the self-monitoring checklist as they complete the task. When they have mastered how to “read” and retell a math word problem, they advance to the organization step. Each step is added successively until they have learned and applied the entire routine. After small group instruction or homework, students are expected to return to the general education math class and use what they have learned about solving math problems. General education teachers must be made aware of the instruction that students are receiving and supplement and support this instruction in the general education math classes.

To do this, it is essential that general and special education teachers communicate regularly about the children and the instruction and coordinate what is taught in the general education class with what is taught in the special education resource class and vice versa. Continuity across general and special education is essential for student success. General education teachers must reinforce what students have learned to ensure that they apply appropriately, and also maintain, acquired skills and strategies.

See page 17 for an example of how a general education teacher and a special educator collaborate on instruction in math problem solving for their second grade students.

Strategy Cues

Students who have a second or third grade reading level are given reminders and prompts such as individual Student Cue Cards to carry with them for home and class use, Master Class Charts on the classroom walls, problem type diagrams and checklists, think-aloud protocols, and discussion about the benefits of using strategies.

The next section presents several instructional procedures that are the foundation of cognitive strategy instruction. These include verbal rehearsal, process modeling, visualization, role reversal, performance feedback, distributed practice, and mastery learning.

Verbal Rehearsal

Before students actually solve problems, they must first learn the steps and memorize them by using verbal rehearsal. This is a memory strategy that enables students to recall automatically the math problem-solving processes and strategies. Students in primary school can learn a SAY, ASK, CHECK routine similar to Montague's (2003) or Jitendra's (2005) as they are learning how to represent problems. Frequently, acronyms are created to help students remember as they verbally rehearse and internalize the labels and definitions for the processes and strategies. For Jitendra's math problem-solving routine, the acronym **FOPS** was created (**F** = Find the problem type, **O** = Organize the information using the change diagram, **P** = Plan to solve the problem, and **S** = Solve the problem. For non-reading students visual prompts can be used as substitutes for letters or along with letters. Cues and prompts are used to help students as they memorize the processes and their definitions. When students have memorized the math problem-solving routine, they can cue other students and the teacher during practice sessions.

Process Modeling

Process modeling is thinking aloud while demonstrating an activity. For mathematical problem solving, this means that the problem solver says everything she or he is thinking and doing while solving a problem. When students are first learning how to apply the processes and strategies, the teacher demonstrates and models what good problem solvers do as they solve problems. Students have the opportunity to observe and hear how to solve mathematical problems. Both correct and incorrect problem-solving behaviors are modeled. Modeling of correct behaviors helps students understand how good problem solvers use the processes and strategies appropriately. Modeling of incorrect behaviors allows students to learn how to use self-regulation strategies to monitor their performance and locate and correct errors. Self-regulation strategies are learned and practiced in the actual context of problem solving. When students learn the modeling routine, they then can exchange places with the teacher and become models for their peers. Initially, students will need plenty of prompting and reinforcement as they become more comfortable with the problem-solving routine. However, they soon become proficient and independent in demonstrating how good problem solvers solve math problems. One of the instructional goals is to gradually move students from overt to covert verbalization. As students become more effective problem solvers, they will begin to verbalize covertly and then internally. In this way, they not only become more effective problem solvers, but they also become more efficient problem solvers.

Visualization

Visualization is critical to problem representation. It allows students to construct an image of the problem on paper or mentally. Students must be shown how to select the important information in the problem and develop a schematic representation. To do this, teachers model how to use manipulatives to represent a problem, and then how to draw a picture or make a diagram that shows the relationships among the problem parts using both the linguistic and numerical information in the problem. These three-dimensional and two-dimensional visual representations can take many forms and will vary from student to student. As students become better problem solvers, they will use a variety of visual representations including manipulatives, pictures, tables, graphs, or other types of displays. Initially, students must be shown how to use the manipulatives and also how to translate the results of their manipulations with concrete objects to more symbolic representations using paper and pencil, e.g., the problem type diagrams. Later, as students become more proficient, they will progress to mental images. Interestingly, if the problem is novel or challenging, they frequently return to conscious application of processes and strategies, which is typical of good problem solvers.

Role Reversal

Role reversal is an important instructional activity that promotes independent learners. As students become familiar with the math problem-solving routine, they can take on the role of teacher as model and actually change places with the teacher. They may use an overhead projector just as the teacher did and engage in process modeling to demonstrate that they can apply effectively the cognitive and metacognitive processes and strategies they have learned. Other students can prompt or ask questions for clarification. In this way, students learn to think about, explain, and justify their visual representations and their solution paths. Teachers may also take the role of the student, guiding the "student as teacher" through the process.

Performance Feedback

Performance feedback is critical to the success of the program. Progress checks are given throughout instruction to determine mastery of the routine. Teachers and parents can assist students with graphing their progress and visually displaying their performance, which is very reinforcing for them. Teachers carefully analyze performance during practice sessions and on mastery checks and provide each student with immediate, corrective feedback. Appropriate use of processes and strategies is reinforced continuously until students become proficient. Students need to know the specific behaviors for which they are praised so they can repeat these behaviors. Praise should be honest. Students should be taught how to give and receive reinforcement and to reinforce themselves, and should have plenty of opportunities to practice doing it. The goal is to teach students to monitor, evaluate, and reinforce themselves as problem solvers.

Distributed Practice

Distributed practice is the cornerstone for ensuring that students maintain what they have learned. To become good math problem solvers, students learn to use the processes and strategies that successful problem solvers use. As a result, their math problem-solving skills and performance levels improve. However, to achieve high performance, students must be given ample opportunity to practice initially as they learn the math problem-solving routine and, then, to maintain high performance, they must continue to practice intermittently over time. They may practice individually or in teams or small groups. They should be involved in solving a range of problems from textbook-type problems to problems encountered in real life.

Mastery Learning

Prior to instruction, a pretest is given to determine baseline performance levels of individual students. During instruction, periodic mastery checks are given to monitor student progress over time and to determine effectiveness of the program. If students are not making sufficient progress, modifications to the program to ensure success must be made. Following instruction, periodic maintenance checks are provided. If students do not meet criterion on maintenance checks, booster sessions must be provided to improve performance levels to mastery. Booster sessions are brief lessons to review and refresh what students have previously learned and mastered.

Teaching Math Problem Solving to Primary School Students with LD

Consider Ms. Gerber's second grade class. She has 24 students in her class. Four have identified learning disabilities and receive resource room support. These students have considerable difficulty solving math word problems. Ms. Gerber notices that another six students are also having difficulty solving math word problems. She decides to seek help from the special education resource teacher, Mr. Rafferty. They decide to work together on improving students' math problem-solving skills. The resource teacher will teach the four students with LD during their resource time. Ms. Gerber will provide small group instruction for the six students from her class who are also having difficulty during independent seatwork time. This coincides with the resource period for the students with LD. They decide to use the routine developed by Jitendra and colleagues. Together, they develop structured lessons and create the necessary cues and prompts. They make a plan to meet every other day for about 20 minutes to identify what is working and what needs improvement in their lessons and also to appraise the progress of individual students. Some of the students are very poor readers. They know that they need to read each problem aloud with the students before having them solve it.

Ms. Gerber and Mr. Rafferty have already progressed through the change problem with a missing ending. They are now ready to start the change problem with a missing change. Their meetings have been very helpful. They have talked through a number of concerns and brainstormed about techniques that will help some of the students who are having some difficulty. All of the students are making progress, although some need additional assistance, which is provided primarily by peers during peer coaching time at the end of the day. Ms. Gerber oversees and monitors the peer coaching. She provides support for poor readers by pairing them with good readers. Below is a structured lesson designed to teach students how to solve the change problem with a missing ending. Ms. Gerber is modeling how good problem solvers solve a change problem with a missing ending. It may be necessary and important to demonstrate solving the problem using an oversized, laminated Change Problem Diagram and manipulatives before demonstrating at this more symbolic level. She places a transparency of the math problem on the projector.

Ms. Gerber: Watch me say everything I am thinking and doing as I solve this problem.

Sara had 5 flowers. Andy gave her some more flowers. Now Sara has 12 flowers. How many flowers did Andy give her?

I am going to start with Step 1 – Find the problem type. Did I read and retell the problem? No, I need to do that. (She reads the problem aloud). Now I will retell the problem. One girl has 5 flowers and her friend gives her more so she has 12 altogether. How many does her friend give her? Okay, I read the problem and said it in my own words. I can check that box. Next, did I ask if it is a change problem? Did I look at the beginning, change, and ending just like the diagram? Do they all describe the same thing? Yes, they do. Flowers.

Okay, on to Step 2 – Organize the information using the change diagram. Did I underline the label that describes the beginning, change, and ending and write in the label in the diagram? I will underline flowers. That is the label. I will write flowers in the beginning and ending circles and in the change box. Did I underline the important information, circle the numbers, and write the numbers in the diagram? I will circle 5 and 12. I will write 5 in the beginning circle and 12 in the ending. I need to find the change. Did I write a "?" for what must be solved? I will write a question mark in the change box. Did I find my question sentence? Yes, the question is, how many does her friend give her?

Now to Step 3 – Plan to solve the problem. Do I add or subtract? I need to subtract 5 from 12 to find how many flowers her friend gave her. Did I write the math sentence? The math sentence is 12 minus 5.

Now, the last step. Step 4 – Solve the problem. Did I solve the math sentence? 12 minus 5 is 7. Did I write the complete answer? The complete answer is 7 flowers. Did I check if the answer makes sense? Yes, the answer makes sense. I am done.

Students then review the Change Problem Diagram and Checklist. As a review, they verbally rehearse the steps and corresponding self-monitoring questions. Then they are given a problem and Ms. Gerber guides them as a group while they think aloud and solve the next problem. A student is then selected to model the process with assistance from Ms. Gerber. Instruction is systematic, sequenced, slow but intense, and structured to ensure mastery as students learn the routine. Students have now learned to solve two variations of the change problem: the missing ending, which requires addition, and the missing change, which requires subtraction. Although Ms. Gerber and Mr. Rafferty were successful in their collaborative effort to help students become better problem solvers and they know the program was effective for their students, they also know that sometimes collaboration can be difficult.

Some Realities of Teaching and Collaborating

- Individualizing instruction may be difficult given the large numbers of students enrolled in most elementary school classes. Class size can range from 20 to 35 students. Enlisting the aid of the resource teacher to assist with individualized and small group instruction may be necessary but not always feasible. One solution may be to hire paraprofessionals or enlist the aid of volunteers to assist classroom teachers. Teachers often have these individuals work with low-performing students. It is preferable to have them monitor and assist higher-performing students, which frees the classroom teacher to work directly with students who are having difficulty. In this way, the teacher can re-teach concepts, skills, and strategies and reinforce what has been taught.
- Identifying the students who need instruction and then grouping for instruction based on the various levels in the class can be a challenge for a math teacher. There are many easily administered formal and informal measures that provide relatively valid performance levels. Additionally, basal series usually include curriculum-based measures that provide measures of baseline performance and progress over time.
- General education math teachers often feel unprepared to teach students who are in special programs. They may not feel confident that students can learn how to think differently and

become good problem solvers. If teachers feel the need for more knowledge and skills in teaching mathematics, particularly problem solving, to students with special needs, they should submit a request in writing to their administrator(s) for professional development training in mathematics. Special educators should attend this training with general educators, as this could be an opportunity to develop a long-term plan for collaboration between general and special education.

- Finding time to talk with the resource teacher for students in special education can be difficult. Also, teachers often do not coordinate resource room instruction with the general education math curriculum. Communication between teachers sometimes can be difficult. Administrators need to be involved in order to set aside time for teachers to discuss students and to coordinate instruction. Special educators should be involved in curriculum meetings, grade level meetings, and, of course, child study team meetings to facilitate communication and coordination.
- Teachers may need to develop the knowledge and skills to implement mathematical problem-solving instruction successfully. Because the program is intense and highly interactive, teachers may need professional development to learn the instructional procedures that are the foundation of cognitive strategy instruction. Again, administrators must be informed about the need for professional development.
- Teachers may not be familiar with the research that supports cognitive strategy instruction, nor familiar with the research that supports cognitive instructional procedures. Professional development should give teachers both a background in research-based practice and also knowledge about its implementation.

Modifying Math Problem-solving Instruction for Students with Other Types of Disabilities

Students with other types of disabilities frequently display cognitive characteristics that resemble those of students with LD. However, their cognitive deficits may be more or less severe or may vary in some unique way from those of students with LD. In many cases, though, there seem to be more similarities than differences. For example, students with spina bifida have long-term memory, visual-spatial, and self-regulation problems that adversely affect their ability to comprehend text and do mathematics (Mesler, 2004). Children with chronic illnesses (such as those surviving cancer) who have undergone intrusive medical treatments often display attention difficulties, short-term memory problems, and other cognitive problems that interfere with school success (Bessell, 2001). Characteristically, students with ADHD have serious self-regulation problems. Students with traumatic brain injury and Asperger's Syndrome also have cognitive deficits that are similar to students with LD. Because of these similarities, it seems reasonable to assume that instruction effective for students with LD may, with modifications, be effective for students with other types of cognitive disabilities.

Conclusion

A systematic, research-based math problem-solving program makes mathematical problem solving easy to teach. Students are provided with the processes and strategies that make math problem solving easy to learn, and they become successful and efficient problem solvers. They also gain a better attitude toward problem solving when they are successful and develop the confidence to persevere. Moving from textbook problems to real life math situations creates a challenge for students, and they begin to understand why they need to be good problem solvers. Cognitive strategy instruction in mathematical problem solving gives students the resources to solve authentic, complex mathematical problems they encounter in everyday life. Teachers who are knowledgeable about the research underlying effective instruction will be able to justify the instructional time spent on small group instruction in math problem solving. They will also be able to explain how the supplemental instruction complements and builds on the mathematics curriculum.

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